Electrical Principles

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1. Introduction
2. Coordinate systems
    A. Rectangular coordinates
        i. Origin, X and Y
            a. **The two numbers that are used to define a point on a graph using rectangular coordinates represent the coordinate values along the horizontal and vertical axes.** [E5C11]
        ii. Complex coordinates
            a. Imaginary numbers *i* and *j*
            b. 1/*j* = -*j*
            c. X + *j*Y
        iii. Resistive and reactive
            a. Horizontal and vertical axes
            b. **In rectangular coordinates, the horizontal axis represents the resistive component.** [E5C09]
            c. **In rectangular coordinates, the vertical axis represents the reactive component.** [E5C10]
            d. **The rectangular coordinate system is often used to display the resistive, inductive, and/or capacitive reactive components of impedance.** [E5C13]
            e. Inductive (top) and capacitive (bottom) Z = R + *j*X
            f. **Capacitive reactance in rectangular notation is represented by –*j*X.** [E5C01]
            g. Oh, I see (OIC) rule
            h. **If you plot the impedance of a circuit using the rectangular coordinate system and find the impedance point falls on the right side of the graph on the horizontal axis, the circuit is equivalent to a pure resistance.** [E5C12]
            i. **Impedance 50 – *j*25 represents 50 ohms resistance in series with 25 ohms capacitive reactance.** [E5C06]
    B. Calculating impedance
        i. $\mathbf{XL} = 2\pi \mathrm{fL}$ to calculate inductor's reactance
            a. **Series circuit consisting of a 300 ohm resistor and an 18 microhenry inductor at 3.505 MHz** (300 + *j*400) (Example 4.4) [E5C15]
        ii. $\mathbf{XC} = \frac{1}{2\pi \mathrm{fC}}$ to calculate capacitor's reactance
            a. **Series circuit consisting of a 400 ohm resistor and a 38 picofarad capacitor at 14 MHz** (400 - *j*300) (Example 4.6) [E5C14]
            b. **Series circuit consisting of a 300 ohm resistor and a 19 picofarad capacitor at 21.200 MHz** (300 - *j*400) (Example 4.7) [E5C16]
            c. **Series circuit consisting of a 300 ohm resistor, a 0.64 microhenry inductor and an 85 picofarad capacitor at 24.900 MHz** (300 + *j*100 - *j*75 = 300 + *j*25) (Example 4.8) [E5C17]
    C. Polar coordinates
        i. Scalars and vectors
        ii. A vector has magnitude and direction
        iii. They're called polar coordinates because it's like looking at the earth from the north pole

- iv. r $\angle\theta$ [or (r, θ)]
- v. **In polar coordinates, a vector has both magnitude and an angular component**. [E5C07]
- vi. Inductive/capacitive reactance on graph
- vii. **Impedances are described by phase angle and amplitude.** [E5C02]
- viii. **The polar coordinate system is often used to display the phase angle of a circuit containing resistance, inductive and/or capacitive resistance.** [E5C08]
- ix. **In polar coordinates, a positive phase angle represents an inductive reactance**. [E5C03]
- x. **In polar coordinates, a negative phase angle represents a capacitive reactance.** [E5C04]
- xi. **A phasor diagram shows the phase relationship between impedances at a given frequency.** [E5C05]

D. Basic trigonometry

- i. Triangles
- ii. Sin, cos, tan, $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$ (cos 30° = 0.866; cos 45° = 0.707; cos 60° = 0.5; tan 45° = 1.0)
- iii. Chief SOHCAHTOA

E. Converting between rectangular and polar coordinates

- i. Polar to rectangular
  - a. r $\angle\theta$ → (a + *j*b) **Example: 10 $\angle 30°$ → ?**
  - b. a = r cos θ = 10*0.866 = 8.7 (rounded)
  - c. b = r sin θ = 10*0.5 = 5 **? = 8.7 + *j*5**
- ii. Rectangular to polar
  - a. (a + *j*b) → r $\angle\theta$ **Example: 3 + *j*3 → ?**
  - b. $r = \sqrt{(a^2 + b^2)}$ = $\sqrt{(3^2 + 3^2)}$ = $\sqrt{18}$ = 4.2 (rounded)
  - c. θ = $\tan^{1}(-1)\,(\frac{b}{a})$ = $\tan^{1}(-1)\,(\frac{3}{3})$ = $\tan^{1}(-1)\,(1)$ = 45° **? = 4.2 $\angle$45°**

3. Electromagnetic fields

A. **E/M field (or electrostatic field) stores potential energy.** [E5D08]

B. **Strength of magnetic field around a conductor is determined by amount of current flowing through the conductor.** [E5D07]

C. **Orientation around the conductor is determined by left hand rule using electron flow (electronic current).** [E5D06]

4. Time constants and phase relationships

A. RLC Time constants [see graph, p. 4-10]

- i. **Time required for the capacitor in an RC circuit to be charged to 63.2% of the applied voltage is one time constant.** [E5B01]
- ii. **Time it takes for a charged capacitor in an RC circuit to discharge to 36.8% of its initial voltage is one time constant.** [E5B02]
- iii. τ = RC [resistance in ohms, capacitance in farads]
- iv. See p. 4-13 for formulas for R_T and C_T, both series and parallel.
- v. **What is the time constant for a circuit having two 220 microfarad capacitors and two 1 megohm resistors, all in parallel? 220 seconds** [E5B04] Example on p. 4-13
- vi. Curve for current buildup in RL circuit is identical to voltage curve for charging capacitor

- B. Phase angles between voltage and current
  - i. See Figure 4.14 on p. 4-15. Remember ICE.
  - ii. **Current through a capacitor leads the applied voltage by 90°.** [E5B09]
  - iii. See Figure 4.17 on p. 4-16. Remember ELI the ICE man.
  - iv. **Voltage applied to an inductor leads the current through it by 90°.** [E5B10]
  - v. **What is the phase angle between the voltage across and the current through a series RLC circuit if XC is 500 ohms, R is 1 kilohm, and XL is 250 ohms? 14 degrees with the voltage lagging the current.** [E5B07]
  - vi. **What is the phase angle between the voltage across and the current through a series RLC circuit if XC is 100 ohms, R is 100 ohms, and XL is 75 ohms? 14 degrees with the voltage lagging the current.** [E5B08]
  - vii. **What is the phase angle between the voltage across and the current through a series RLC circuit if XC is 25 ohms, R is 100 ohms, and XL is 50 ohms? 14 degrees with the voltage leading the current.** [E5B11]

5. Admittance and susceptance
   - A. Measured in Siemens (S)
   - B. Reciprocals (inverse)
     - i. Resistance → conductance (G)
     - ii. **Impedance → admittance (Y)** [E5B12]
     - **iii. Reactance → susceptance (B)** [E5B06]
     - iv. **The letter B is commonly used to represent susceptance.** [E5B13]
     - v. These are also reciprocals in magnitude
     - vi. **What happens to the magnitude of a reactance when it is converted to a susceptance? The magnitude of the susceptance is the reciprocal of the magnitude of the reactance.** [E5B05]
     - vii. Change sign of a phase angle when taking its reciprocal
     - viii. **What happens to a phase angle of a reactance when it is converted to a susceptance? The sign is reversed.** [E5B03]
     - ix. Reciprocals can also be written in either rectangular or polar form
6. Reactive power and power factor
   - A. Reactive power
     - i. Resistive part of the circuit consumes and dissipates power as heat
     - ii. **In an AC circuit with ideal inductors and capacitors, reactive power is repeatedly exchanged between the associated magnetic and electric fields, but is not dissipated.** [E5D09]
     - iii. **Reactive power is therefore wattless, nonproductive power.** [E5D14]
   - B. Power factor
     - i. PF = P_{REAL} / $P_{APPARENT}$ and ranges from 0 to 1. Power is consumed by resistance.
       - a. If PF =1, voltage & current are in phase and all apparent power is real power
       - b. If PF = 0, voltage & current are 90° out of phase and all apparent power is reactive power
     - ii. P_{REAL} = $P_{APPARENT}$ X PF
       - a. **How can the true power be determined in an AC circuit where the voltage and current are out of phase? By multiplying the apparent power times the power factor.** [E5D10]
       - b. (Example 4.16 on p. 4-28:) **How many watts are consumed in a circuit having a power factor of 0.71 if the apparent power is 500 VA? 355 W** [E5D18]

    iii. Phase angle: PF = $\cos\theta$
        a. **What is the power factor of an R-L circuit having a θ phase angle? [cos 30° = 0.866; cos 45° = 0.707; cos 60° = 0.5]** [E5D11, E5D15, E5D16]
    iv. P = I E (current times voltage)
        a. (Example 4.13 on p. 4-27:) **How many watts are consumed in a circuit having a power factor of 0.2 if the input is 100 VAC at 4 amperes? 80 watts** [E5D12]
        b. (Example 4.15 on p. 4-28:) **How many watts are consumed in a circuit having a power factor of 0.6 if the input is 200 VAC at 5 amperes? 600 watts** [E5D17]
    v. $P = I^2R$ for a series circuit, where I is RMS current [Twinkle, twinkle, little star]
        a. (Example 4.14 on p. 4-27:) **How much power is consumed in a circuit consisting of a 100 ohm resistor in series with a 100 ohm inductive reactance drawing 1 ampere? 100 watts** [E5D13]
    vi. $P = E^2/R$ for a parallel circuit, where E is the RMS voltage

7. Skin effect and conductor length
    A. Skin effect due to frequency increase
        i. With frequency increase, EM fields of signals don't penetrate as deeply into a conductor such as a wire
        ii. At dc, the whole cross-section of the wire carries current, but with **increasing frequency, the current is confined to regions closer and closer to the surface of the wire, shrinking the effective area.** [E5D01]
    B. Conductor length
        i. Want to avoid unexpected and unwanted **parasitic inductance** in leads, **increases with frequency.** [E5D05]
        ii. Even #24 AWG wire has an inductance of about 20 nH per inch of length
        iii. **To avoid unwanted inductive reactance in circuits used for VHF and above, keep lead lengths short.** [E5D02]
        iv. **At microwave frequencies, short leads reduce phase shifts along the connection.** [E5D04]
        v. **Microstrip are precision printed circuit conductors above a ground plane that provide constant impedance interconnects at microwave frequencies.** [E5D03]
8. Resonant circuits
    A. Resonant frequency
        i. **Resonance in an electrical circuit is the frequency at which the capacitive reactance equals the inductive reactance.** [E5A02]
        ii. Circuit is resonant when the inductive reactance value is the same as the capacitive reactance value, regardless of series or parallel
        iii. **Resonance can cause the voltage across reactances in series to be larger than the voltage applied to them.** [E5A01]
        iv. Because the voltages across the inductor and capacitor cancel each other out, **the magnitude of the impedance of a series (or parallel) RLC circuit at resonance is approximately equal to the circuit resistance.** [E5A03, E5A04]
        v. Resonant frequency: $f_r = \frac{1}{2\pi\sqrt{LC}}$
        vi. Example following equation 4.13 on p. 4-29. Note exponents and resistance. **What is the resonant frequency of a series RLC circuit if R is 22 ohms, L is 50 microhenries, and C is 40 picofarads? 3.56 MHz.** [E5A14]

    vii. Example at top of p. 4-30. **What is the resonant frequency of a parallel RLC circuit if R is 33 ohms, L is 50 microhenries, and C is 10 picofarads? 7.12 MHz.** [E5A16]
  B. Impedance vs. Frequency
    i. As mentioned earlier, at resonance, impedance approximates the circuit resistance for both series and parallel circuits
    ii. **For** both **series** and parallel **RLC circuits at resonance**, because inductive and capacitive reactance are equal but opposite, they cancel each other out, and resulting **current and voltage are in phase.** [E5A08]
    iii. **Current reaches maximum at resonant frequency at the input of a series-resonant circuit**. [E5A05]
    iv. **For a parallel RLC circuit at resonance, the input current is minimum,** [E5A07] **but the circulating current is maximum** [E5A06] because of energy exchange between inductor and capacitor
9. Q (Quality factor)
  A. Q = X/R
    i. One definition of Q is the ratio of reactance to resistance in a circuit
    ii. **The lower the** resistive **losses, the higher the Q.** [E5A15]
  B. $Q_{SERIES} = \frac{1}{R}\sqrt{\frac{L}{C}}$ or Q_{SERIES} **= X/R**
    i. **How is the Q of an RLC series resonant circuit calculated? Reactance of either the inductance or capacitance divided by the resistance.** [E5A10]
  C. $Q_{PARALLEL} = R\sqrt{\frac{C}{L}}$ or $Q_{PARALLEL}$ **= R/X**
    i. **How is the Q of an RLC parallel resonant circuit calculated? Resistance divided by the reactance of either the inductance or capacitance.** [E5A09]
  D. **As Q increases in a resonating circuit, so do internal voltages and circulating currents.** [E5A13]
10. Half-power bandwidth
  A. Describe half-power points f1 and f2 in Figure 4.29 on p. 4-33
  B. Half-power bandwidth Δf = difference between f1 and f2
  C. The higher the Q, the sharper the resonant circuit's frequency response
  D. Half-power bandwidth $\Delta f = \frac{f_r}{Q}$
  E. Examples at bottom of p. 4-33 and top of 4-34
    i. **What is the half-power bandwidth of a parallel resonant circuit that has a resonant frequency of 7.1 MHz and a Q of 150? 47.3 kHz.** [E5A11]
    ii. **What is the half-power bandwidth of a parallel resonant circuit that has a resonant frequency of 3.7 MHz and a Q of 118? 31.4 kHz.** [E5A12]
11. Impedance matching circuits (p. 4-34)
  A. Transform one ratio of voltage to current (impedance) at the output to another at the input
  B. Again, as Q increases, internal voltages and currents increase
  C. **As Q increases, matching bandwidth decreases,** [E5A17] like that of a resonant circuit